Stochastic dynamics near a change of stability (Amplitude- and Modulation-Equations)

Abstract: Modulation- or Amplitude-Equations are a universal tool to approximate solutions of complicated systems like partial or stochastic partial differential equations (SPDEs) near a change of stability, when there is no center manifold theory available. One can rely on the natural separation of time-scales at the bifurcation to show that the solution of the original equation is well described by the bifurcating pattern with an amplitude that is slowly modulated in time and also in space, if the underlying domain is sufficiently large. This amplitude satisfies an equation on the slow time- and space-scale, which is called Amplitude- or Modulation-Equation.

This is useful to explain qualitatively noise induced pattern formation below the change of stability and stabilization (i.e. destruction of pattern) due to degenerate additive noise. The approach is on a formal level well known in the physics literature, and for partial differential equations on unbounded domains rigorously studied in the last two decades. Although the results for stochastic equations are far more general, for simplicity of presentation we focus mostly on the less technical stochastic Swift-Hohenberg equation and as an example the convective instability in Rayleigh-Benard convection.